Why we don't see the Schrödinger's cat state?

Toshifumi Sakaguchi

ASCII Corporation

Yoyogi 4-33-10, Shibuya-Ku, Tokyo, Japan

(February 1, 2008)

Abstract

Schrödinger's cat puzzle is resolved. The reason why we do not see a macroscopic superposition of states is cleared in the light of Everett's formulation of quantum mechanics.

03.65.Bz

Schrödinger argued [1,2] incompleteness of the Copenhagen interpretation of quantum mechanics by showing a paradoxical thought experiment known as "Schrödinger's cat": A cat is placed in a sealed box with a device which releases a fatal dose of cyanide if a radioactive decay is detected. After a while, a human opens the box to see if the cat is alive or dead. According to the Copenhagen interpretation, the cat was neither alive nor dead until the box was opened, and when the human peeked in the box the wavefunction of the cat collapsed into one of the two alternatives (otherwise the human might be able to see a live cat after having seen a dead cat!). The paradox is that the cat presumably knew if it was alive *before* the box was opened, that is, the cat must have been either alive or dead *before* the human opened the box.

As long as we adhere to the Copenhagen interpretation, the paradox will not be resolved scientifically, because it provides no means to analyze the whole system including the observer system. In the Copenhagen interpretation, the measurement process cannot be described by any equation, but instead must be implemented by hand.

Everett proposed a new formulation of quantum mechanics [3], in which the collapsing process of the wavefunction is ascribed to a branching process of the observer state. That is, the wavefunction of the cat never collapses into one of the two alternatives but instead the wavefunction of the human branches into two states, each of which being correlated to one of the alternatives. One then may ask why the observer state always correlates to the live-cat state and the dead-cat state, rather than the other possible superposed alternatives, or to the "Schrödinger's cat state". Recent studies [4–13] have been focused on this issue and people have tried to show how the preferred basis can be selected through the process of interaction with the environment.

In this Letter, I will give an answer to this puzzle from a different point of view. In order to understand what it means, we need to refine the formulation. The answer will then follow as a natural consequence.

In Everett's formulation of quantum mechanics, we do *not* interpret the squared modulus of the wavefunction as the probability, but interpret it as a *measure* which satisfies the

following property: When a subset of a superposition $\sum_i X_i|\xi^i\rangle$ is described as a single element $Y|\eta\rangle$;

$$Y|\eta\rangle = \sum_{i\in\mathcal{I}} X_i|\xi^i\rangle, \tag{1}$$

then the corresponding measure m must satisfy the relation;

$$m(Y) = \sum_{i\in\mathcal{I}} m(X_i). \tag{2}$$

In Eq. (1), the elements in the set $\{|\xi^i\rangle\}$ have no overlap with each other, that is, $\langle\xi^i|\xi^j\rangle = 0$ if $i \neq j$. To distinguish the coefficients from the states uniquely, we require that the states themselves are always normalized and that the measure is a function of the absolute value of the coefficients. Then the function m can be determined uniquely [3]:

$$m(|X_i|) = |X_i|^2. \tag{3}$$

This is all about Everett's formulation of quantum mechanics. It has nothing to do with the probability.

On the other hand, *if one is interested in the measurement process*, the conventional probabilistic interpretation can be *derived* as follows.

We describe an observer state in which an observed value α^i, one of the eigenvalues of an observable α, is recoded, as $|[\alpha^i]\rangle$. When we observe an object system prepared in an *eigenstate* $|\alpha^i\rangle$, we always get the value α^i. Therefore, the measurement process will be described:

$$U(t)|\alpha^i\rangle|[\,]\rangle = |\alpha^i\rangle|[\alpha^i]\rangle, \tag{4}$$

where $U(t)$ is a time evolution operator obtained from a Hamiltonian which includes the interaction between the object and observer systems.

We now prepare N identical object systems in a same state $|\psi\rangle = \sum_{i=1}^{M} C_i|\alpha^i\rangle$, and observe α for each object sequentially. Applying Eq. (4) to each measurement process sequentially, we will get a *superposition* of branched states of the form;

$$C_pC_q\dots C_r|\alpha^p\rangle|\alpha^q\rangle\dots|\alpha^r\rangle|[\alpha^p\alpha^q\dots\alpha^r]\rangle. \tag{5}$$

Measure $m(n_1, n_2, \dots, n_M)$ assigned to a subset of the superposition of the branched states where α^1 is recorded n_1 times, α^2 is recorded n_2 times, ..., and α^M is recorded n_M times in each memory [...] is calculated as follows. Each state vector described above has the measure $|C_1|^{2n_1}|C_2|^{2n_2}\dots|C_M|^{2n_M}$ and there are $N!/\prod_{i=1}^M n_i!$ states in the subset, hence we get,

$$m(n_1, n_2, \dots, n_M) = \frac{N!}{\prod_{i=1}^M n_i!}\prod_{i=1}^M |C_i|^{2n_i}, \tag{6}$$

where

$$N = \sum_{i=1}^M n_i. \tag{7}$$

A subset which gives a maximum value of the measure $m(n_1, n_2, \dots, n_M)$ in the limit $N \to \infty$ can be obtained from the following equations:

$$\begin{aligned} F &= \ln m(n_1, n_2, \dots, n_M) - \lambda(N - \sum_{i=1}^M n_i) \\ &\simeq N\ln N - N - \sum_{i=1}^M (n_i \ln n_i - n_i) + \sum_{i=1}^M n_i \ln |C_i|^2 \\ &- \lambda(N - \sum_{i=1}^M n_i), \end{aligned} \tag{8}$$

and

$$\frac{\partial F}{\partial n_i} = 0 \qquad (i = 1, 2, \dots, M), \tag{9}$$

$$\frac{\partial F}{\partial \lambda} = 0. \tag{10}$$

From Eq. (9) we get $n_i = e^\lambda|C_i|^2$, and Eq. (10) yields $N = e^\lambda\sum_{i=1}^M |C_i|^2 = e^\lambda$, hence we obtain,

$$\frac{n_i}{N} = |C_i|^2. \tag{11}$$

In the limit $N \to \infty$, the measure R_N assigned to the remaining subset, to which the above elements do not belong, vanishes:

$$
\begin{aligned}
\lim_{N\to\infty} R_N &= \lim_{N\to\infty}\{1 - m(N|C_1|^2, N|C_2|^2, \ldots, N|C_M|^2)\} \\
&= \lim_{N\to\infty}\{1 - \frac{N!}{\prod_{i=1}^{M} n_i!}\prod_{i=1}^{M}(\frac{n_i}{N})^{n_i}\} \\
&= 0. \qquad (12)
\end{aligned}
$$

Therefore, in almost all elements of the superposition, the observer who is described in *each* element will conclude that the eigenvalue α^i can be obtained with probability $n_i/N = |C_i|^2$ *in his world*.

One may attempt to require the disappearance of coherence [14–21] among branched observer states, i.e., that the observer states *must* be orthogonal to each other. However, the coherence cannot be recognized by the observer described by each branched observer state in *principle*. This is guaranteed by the linearity of the time evolution operator. Even though there *is* a moment when the states are coherent because of the *continuous* measurement process, we do not notice the branching process. Therefore, it is superfluous to add the extra condition to complete the derivation.

We now give an answer to the Schrödinger's cat puzzle. Let $|\gamma_{live}\rangle$ and $|\gamma_{dead}\rangle$ be the live-cat state and the dead-cat state, respectively. Note that these vectors are not necessarily eigenvectors of an operator, but are two solutions of Schrödinger's equation.

When the cat is in the *definite* state $|\gamma_{live}\rangle$ *or* $|\gamma_{dead}\rangle$, rather than a superposition of the two states, we always find the live cat *or* dead cat when we peek in the box. That is, if the cat state is $|\gamma_{live}\rangle$ then we always see the live cat and if it is $|\gamma_{dead}\rangle$ then we always see the dead cat. Therefore, the observation process will be described by;

$$
U(t)|\gamma_{live}\rangle|[\,]\rangle = |\gamma_{live}\rangle|[\gamma_{live}]\rangle, \qquad (13)
$$

and

$$
U(t)|\gamma_{dead}\rangle|[\,]\rangle = |\gamma_{dead}\rangle|[\gamma_{dead}]\rangle. \qquad (14)
$$

From Eqs. (13) and (14) and from the linearity of the time evolution operator $U(t)$, it follows that

$$
\begin{aligned}
&U(t)(a|\gamma_{live}\rangle + b|\gamma_{dead}\rangle)|[\,]\rangle \\
&= a|\gamma_{live}\rangle|[\gamma_{live}]\rangle + b|\gamma_{dead}\rangle|[\gamma_{dead}]\rangle,
\end{aligned} \tag{15}
$$

where a and b are non-zero complex numbers. It means that when the cat is in a superposition;

$$
a|\gamma_{live}\rangle + b|\gamma_{dead}\rangle, \tag{16}
$$

and we look in the box, *our state* $|[\,]\rangle$ branches into two states $|[\gamma_{live}]\rangle$ and $|[\gamma_{dead}]\rangle$. Note that the observer states in Eqs. (13) and (14), where there is no other alternative, and those in Eq. (15), which appear in the superposition, are *exactly* the same. This is the reason why we cannot recognize the branching process. One may try to investigate the details of the observer states to see if the observer states can be considered as representing ourselves, or if the observer described by the observer states can be thought of as having a consciousness. However, it will be neither practicable nor indispensable. We only have to know that the observer states which correspond to "observer sees live cat" and "observer sees dead cat" are characterized by Eqs. (13) and (14), respectively. It should be noted that the *measure* assigned to each observer state in Eq. (15) is invariant under any basis transformations.

On the other hand, if we can see the superposition of states *as it is*, the observation process must be described:

$$
\begin{aligned}
&U'(t)(a|\gamma_{live}\rangle + b|\gamma_{dead}\rangle)|[\,]\rangle \\
&= (a|\gamma_{live}\rangle + b|\gamma_{dead}\rangle)|[\Phi_+]\rangle,
\end{aligned} \tag{17}
$$

and

$$
\begin{aligned}
&U'(t)(a|\gamma_{live}\rangle - b|\gamma_{dead}\rangle)|[\,]\rangle \\
&= (a|\gamma_{live}\rangle - b|\gamma_{dead}\rangle)|[\Phi_-]\rangle.
\end{aligned} \tag{18}
$$

From Eqs. (17) and (18) and from the linearity of the time evolution operator $U'(t)$, it follows that

$$U'(t)|\gamma_{live}\rangle|[\,]\rangle = \tfrac{1}{2a}\{(a|\gamma_{live}\rangle + b|\gamma_{dead}\rangle)|[\Phi_+]\rangle$$
$$+ (a|\gamma_{live}\rangle - b|\gamma_{dead}\rangle)|[\Phi_-]\rangle\}, \tag{19}$$

and

$$U'(t)|\gamma_{dead}\rangle|[\,]\rangle = \tfrac{1}{2b}\{(a|\gamma_{live}\rangle + b|\gamma_{dead}\rangle)|[\Phi_+]\rangle$$
$$- (a|\gamma_{live}\rangle - b|\gamma_{dead}\rangle)|[\Phi_-]\rangle\}. \tag{20}$$

Eqs. (19) and (20) show that we cannot see the live cat or the dead cat *even when the cat is in the definite state* $|\gamma_{live}\rangle$ *or* $|\gamma_{dead}\rangle$. It will be possible to realize such an interaction only in *principle*.

The situation described above demonstrates Bohr's complementarity [22], which was *derived* as a theorem in another paper [23].

REFERENCES

[1] E. Schrödinger, Naturwiss. **23**, 807, 823, 844 (1935).

[2] For a review, see *Quantum Theory and Measurement, Edited by J. A. Wheeler and W. H. Zurek*, Princeton University Press (1983).

[3] H. Everett III, Rev. Mod. Phys. **29**, 454 (1957).

[4] H. D. Zeh, Found. Phys. **3**, 109 (1973).

[5] W. H. Zurek, Phys. Rev. D **24**, 1516 (1981).

[6] W. H. Zurek, Phys. Rev. D **26**, 1862 (1982).

[7] A. O. Caldeira and A. J. Leggett, Physica **A 121**, 587 (1983).

[8] W. H. Zurek, in *Proceedings of the International Symposium on the Foundations of Quantum Mechanics in the Light of New Technology*, Tokyo, Japan, 1983, edited by S. Kamefuchi *et. al.* (Physical Society of Japan, Tokyo, 1984), p.181.

[9] W. H. Zurek, *Frontiers of Nonequilibrium Statistical Physics* (Plenum, New York, 1986).

[10] W. Unruh and W. H. Zurek, Phys. Rev. D **40**, 1071 (1989).

[11] W. H. Zurek, Phys. Today **44** (10), 36 (1991).

[12] B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. **D 45**, 2843 (1992).

[13] A. Albrecht, Phys. Rev. D **46**, 5504 (1992).

[14] R. B. Griffiths, J. Stat. Phys. **36**, 219 (1984).

[15] R. Omnès, Ann. Phys. **201**, 354 (1990).

[16] M. Gell-Mann and J. B. Hartle, in *Complexity, Entropy and the Physics of Information, SFI Studies in the Sciences of Complexity*, Vol. VIII, edited by W. H. Zurek (Addison Wesley, Reading, 1990).

[17] J. B. Hartle, in *Quantum Cosmology and Baby Universes*, edited by S. Coleman, J. Hartle, T. Piran and S. Weinberg (World Scientific, Singapore, 1991).

[18] R. Omnès, Rev. Mod. Phys. **64**, 339 (1992).

[19] J. B. Hartle, in Proceedings of the 1992 Les Houches Summer School, *Gravitation et Quantifications*, edited by B. Julia.

[20] M. Gell-Mann and J. B. Hartle, Phys. Rev. **D47**, 3345 (1993).

[21] R. B. Griffiths, Phys. Rev. Lett. **70**, 2201 (1993).

[22] N. Bohr, *Atomic Theory and Description of Nature*, (Cambridge, 1961).

[23] T. Sakaguchi, Submitted to Phys. Rev. Lett.